Objectives:

- Find local maxima and minima of a function graphically.
- Find the absolute maximum and the absolute minimum of a function graphically.
- Use the first derivative test to find local extrema analytically.

Definitions:

- CRITICAL NUMBER: A number c in the domain of a function f is a **critical number** of f if

  $f'(c) = 0$ $f'(c)$ does not exist.

- ABSOLUTE MAXIMUM VALUE: For a number c in the domain of a function f, the number $f(c)$ is the **absolute maximum value** of $f(x)$ if

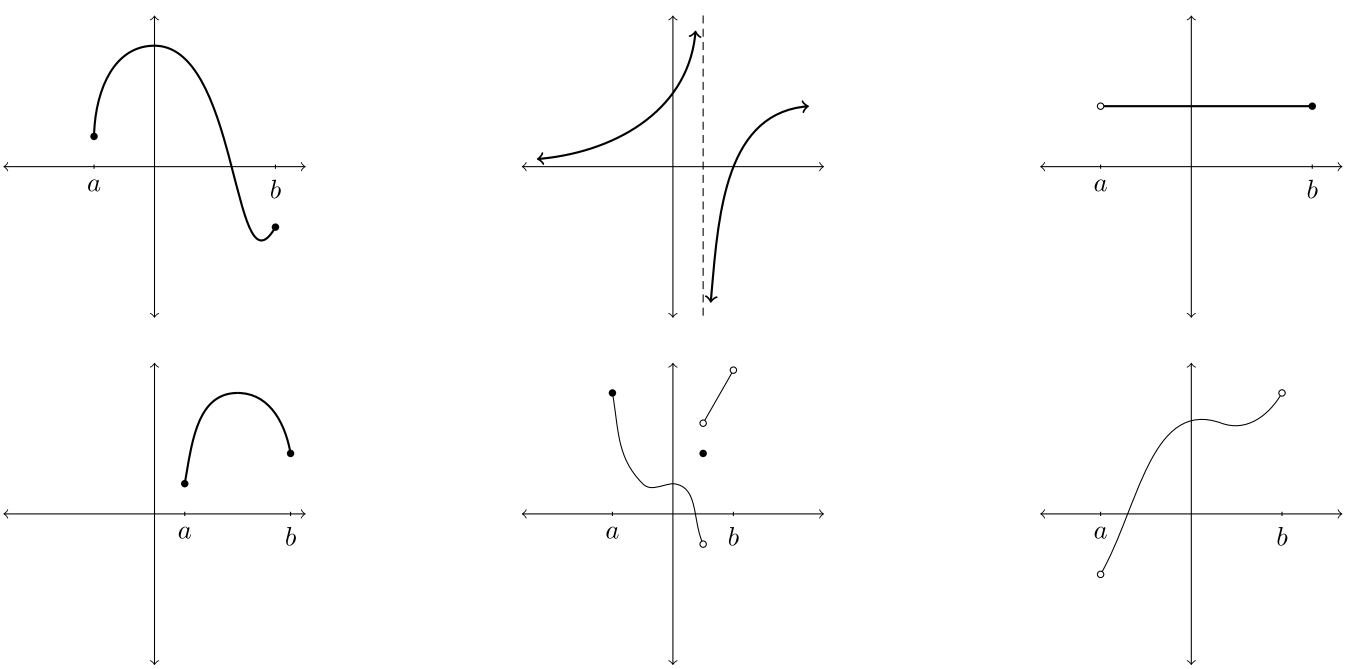
  $$f(c) \geq f(x) \text{ for all } x \text{ in the domain of } f.$$

- ABSOLUTE MINIMUM VALUE: For a number c in the domain of a function f, the number $f(c)$ is the **absolute minimum value** of $f(x)$ if

  $$f(c) \leq f(x) \text{ for all } x \text{ in the domain of } f.$$

We refer to absolute maximum and minimum values as _____absolute extrema_____ .

Functions don't always have absolute extrema. Which of these functions have absolute extrema?



You will work more with absolute extrema in your project tomorrow!

Definitions:

- LOCAL MAXIMUM VALUE: The number $f(c)$ is a **local maximum value** of $f(x)$ if

$$f(c) \geq f(x) \text{ when } x \text{ is near } c.$$

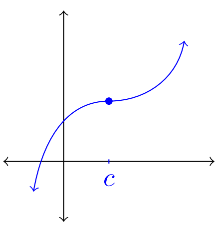
- LOCAL MINIMUM VALUE: The number $f(c)$ is a **local minimum value** of $f(x)$ if

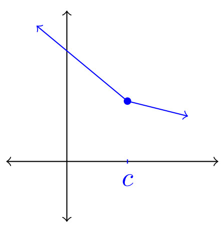
$$f(c) \leq f(x) \text{ when } x \text{ is near } c.$$

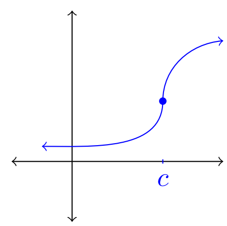
We refer to local maximums and minimums as ________local extrema________ .
If we are on a hunt for local extrema, where should we look?

> **Fermat's Theorem:**
>
> If f has a local maximum or minimum at c, then either $f'(c) = 0$ of $f'(c)$ does not exist (i.e., c is a critical number).

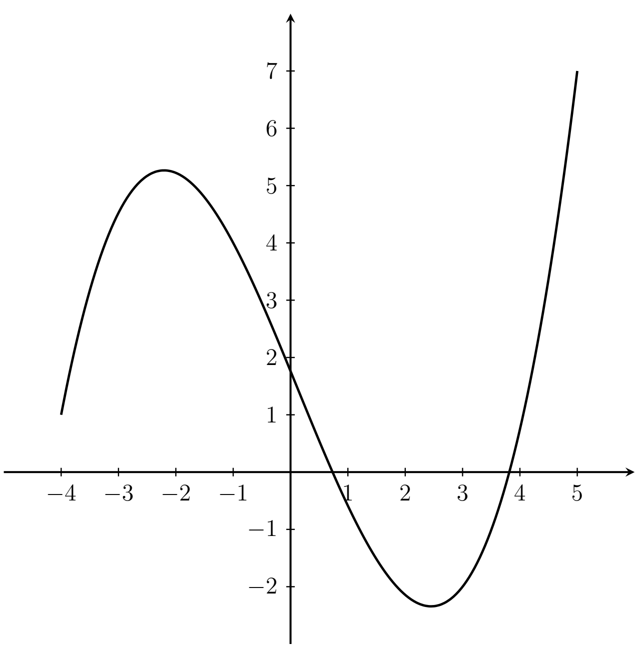
CAUTION: the converse is not true! If f has a critical number c, f doesn't necessarily have a local extreme value there:







Graphical example:



- f is defined on $[-4, 5]$.
- At $x = 5$, f has an absolute max of 7.
- At $x = 2.5$, f has a local min of about -2.25 (also an absolute min).
- At $x = -2.5$, f has a local max of 5 (not an absolute max).

So, if c is a critical number for f, how do we tell if f has a local max, local min, or neither?

Analytically, there are two options (described in Section 4.3). Here is the first one:

First Derivative Test: If c is a critical number for a continuous function f,

(a) If $f'(x)$ changes from positive to negative at c, then f has a local max at $x = c$.

(b) If $f'(x)$ changes from negative to positive at c, then f has a local min at $x = c$.

(c) If $f'(x)$ does not switch signs, then f has no local extremum at $x = c$.

Example: Find and classify all critical numbers of

$$f(x) = 2x^3 + 3x^2 - 12x.$$

Steps:

1. Find $f'(x)$.

$$f'(x) = 6x^2 + 6x - 12$$

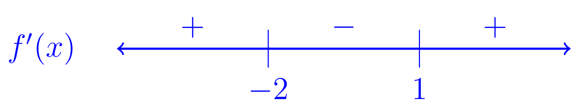
2. Find where $f'(x) = 0$ or where $f'(x)$ DNE.

$$0 = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x - 1)(x + 2)$$

So $x = 1$ and $x = -2$ are critical numbers.

There are no other critical numbers because $f'(x)$ is always defined.

3. Make a sign chart for $f'(x)$.



4. Use the first derivative test to classify each critical number.

At $x = -2$, the derivative switches from positive to negative so the first derivative test tells us that $f(x)$ has a local max at $x = -2$. The local maximum value is $f(-2) = 20$.

At $x = 1$, the derivative switches from negative to positive so the first derivative test tells us that $f(x)$ has a local min at $x = 1$. The local minimum value is $f(1) = -7$.